

Cambridge International AS & A Level

CANDIDATE
NAME

--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

MATHEMATICS

9709/32

Paper 3 Pure Mathematics 3

May/June 2022

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

BLANK PAGE

- 1** Solve the equation $\ln(e^{2x} + 3) = 2x + \ln 3$, giving your answer correct to 3 decimal places. [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

2 Solve the equation $3 \cos 2\theta = 3 \cos \theta + 2$, for $0^\circ \leq \theta \leq 360^\circ$.

[5]

[illegible]

- 3** The polynomial $ax^3 + x^2 + bx + 3$ is denoted by $p(x)$. It is given that $p(x)$ is divisible by $(2x - 1)$ and that when $p(x)$ is divided by $(x + 2)$ the remainder is 5.

Find the values of a and b .

[5]

[illegible]

- 4 The equation of a curve is $y = \cos^3 x \sqrt{\sin x}$. It is given that the curve has one stationary point in the interval $0 < x < \frac{1}{2}\pi$.

Find the x -coordinate of this stationary point, giving your answer correct to 3 significant figures. [6]

[illegible]

- 5 (a) By sketching a suitable pair of graphs, show that the equation $\ln x = 3x - x^2$ has one real root. [2]

- (b) Verify by calculation that the root lies between 2 and 2.8. [2]

.....

.....

.....

.....

.....

.....

.....

- (c) Use the iterative formula $x_{n+1} = \sqrt{3x_n - \ln x_n}$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

.....

.....

.....

.....

.....

.....

.....

- 6 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = xe^{y-x},$$

and $y = 0$ when $x = 0$.

- (a) Solve the differential equation, obtaining an expression for y in terms of x . [7]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- [illegible]

7 The equation of a curve is $x^3 + 3x^2y - y^3 = 3$.

(a) Show that $\frac{dy}{dx} = \frac{x^2 + 2xy}{y^2 - x^2}$. [4]

[illegible]

- (b)** Find the coordinates of the points on the curve where the tangent is parallel to the x -axis. [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings on the page.

8 Let $f(x) = \frac{x^2 + 9x}{(3x - 1)(x^2 + 3)}$.

(a) Express $f(x)$ in partial fractions.

[5]

[illegible]

- (b)** Hence find $\int_1^3 f(x) \, dx$, giving your answer in a simplified exact form. [5]

[illegible]

9 The lines l and m have vector equations

$$\mathbf{r} = -\mathbf{i} + 3\mathbf{j} + 4\mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} - \mathbf{k}) \quad \text{and} \quad \mathbf{r} = 5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k} + \mu(a\mathbf{i} + b\mathbf{j} + \mathbf{k})$$

respectively, where a and b are constants.

(a) Given that l and m intersect, show that $2b - a = 4$.

[4]

[illegible]

- (b) Given also that l and m are perpendicular, find the values of a and b . [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (c) When a and b have these values, find the position vector of the point of intersection of l and m . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- 10** The complex number $-1 + \sqrt{7}i$ is denoted by u . It is given that u is a root of the equation

$$2x^3 + 3x^2 + 14x + k = 0,$$

where k is a real constant.

- (a)** Find the value of k .

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b)** Find the other two roots of the equation.

[4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (c) On an Argand diagram, sketch the locus of points representing complex numbers z satisfying the equation $|z - u| = 2$. [2]

- (d) Determine the greatest value of $\arg z$ for points on this locus, giving your answer in radians. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

[illegible]

BLANK PAGE

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.